

Quantum decoherence and classical correlations in open systems

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Abstract

In the framework of the Lindblad theory for open quantum systems we determine the degree of quantum decoherence and classical correlations of a harmonic oscillator interacting with a thermal bath. The transition from quantum to classical behaviour of the considered system is analyzed and it is shown that the classicality takes place during a finite interval of time. We calculate also the decoherence time and show that it has the same scale as the time after which thermal fluctuations become comparable with quantum fluctuations.

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1 Introduction

The transition from quantum to classical physics and classicality of quantum systems continue to be among the most interesting problems in many fields of physics, for both conceptual and experimental reasons [1, 2, 3]. Two conditions are essential for the classicality of a quantum system [4]: a) quantum decoherence (QD), that means the irreversible, uncontrollable and persistent formation of a quantum correlation (entanglement) of the system with its environment [5], expressed by the damping of the coherences present in the quantum state of the system, when the off-diagonal elements of the density matrix of the system decay below a certain level, so that this density matrix becomes approximately diagonal and b) classical correlations (CC), expressed by the fact that the Wigner function of the quantum system has a peak which follows the classical equations of motion in phase space with a good degree of approximation, that is the quantum state becomes peaked along a classical trajectory. The necessity and sufficiency of both QD and CC as conditions of classicality are still a subject of debate. Both these conditions do not have an universal character, so that they are not necessary for all physical models. An important role in this discussion plays the temperature of the environment and therefore it is worth to take into account the

differences between low and high temperature regimes. For example, purely classical systems at very high temperatures are described by a classical Fokker-Planck equation which does not follow any trajectory in phase space (for very small kinetic energy, compared to the thermal energy, when the probability distribution becomes essentially independent of momentum), so that in this case CC are not necessary. Likewise, one can have a classical behaviour if the coherences are negligible, without having strong CC (for example, in the case of a classical gas at finite temperature) and the lack of strong correlations between the coordinate and its canonical momentum does not necessarily mean that the system is quantum. On the other hand, the condition of CC is not sufficient for a system to become classical – although the Wigner function can show a sharp correlation in phase space, the quantum coherence never vanishes for a closed system which has a unitary evolution. Likewise, in the low temperature quantum regime one can observe strong CC. For example, in the case of a purely damped quantum harmonic oscillator (at zero temperature), the initial coherent states remain coherent and perfectly follow classical trajectories of a damped oscillator, but CC are not sufficient for classicality.

In the last two decades it has become more and more clear that the classicality is an emergent property of open quantum systems, since both main features of this process – QD and CC – strongly depend on the interaction between the system and its external environment [3, 6, 7]. The main purpose of this work is to study QD and CC for a harmonic oscillator interacting with an environment in the framework of the Lindblad theory for open quantum systems. We determine the degree of QD and CC and the possibility of simultaneous realization of QD and CC for a system consisting of a harmonic oscillator in a thermal bath. It is found that the system manifests a QD which increases with time and temperature, whereas CC are less and less strong with increasing time and temperature.

2 Lindblad master equation for the harmonic oscillator in coordinate and Wigner representation

The irreversible time evolution of an open system is described by the following general quantum Markovian master equation for the density operator $\rho(t)$ [8]:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar} \sum_j ([V_j \rho(t), V_j^\dagger] + [V_j, \rho(t) V_j^\dagger]). \quad (1)$$

H is the Hamiltonian of the system and V_j, V_j^\dagger are operators on the Hilbert space of H , which model the environment. In order to obtain, for the damped quantum harmonic oscillator, equations of motion as close as possible to the classical ones, the two possible operators V_1 and V_2 are taken as linear polynomials in coordinate q and momentum p [9, 10] and the harmonic oscillator Hamiltonian H is chosen of the general quadratic form

$$H = H_0 + \frac{\mu}{2}(qp + pq), \quad H_0 = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}q^2. \quad (2)$$

With these choices the master equation (1) takes the following form [9, 10]:

$$\begin{aligned} \frac{d\rho}{dt} = & -\frac{i}{\hbar}[H_0, \rho] - \frac{i}{2\hbar}(\lambda + \mu)[q, \rho p + p\rho] + \frac{i}{2\hbar}(\lambda - \mu)[p, \rho q + q\rho] \\ & - \frac{D_{pp}}{\hbar^2}[q, [q, \rho]] - \frac{D_{qq}}{\hbar^2}[p, [p, \rho]] + \frac{D_{pq}}{\hbar^2}([q, [p, \rho]] + [p, [q, \rho]]). \end{aligned} \quad (3)$$

In the particular case when the asymptotic state is a Gibbs state $\rho_G(\infty) = e^{-\frac{H_0}{kT}}/\text{Tre}^{-\frac{H_0}{kT}}$, the quantum diffusion coefficients D_{pp}, D_{qq}, D_{pq} and the dissipation constant λ satisfy the relations [9, 10]

$$D_{pp} = \frac{\lambda + \mu}{2}\hbar m\omega \coth \frac{\hbar\omega}{2kT}, \quad D_{qq} = \frac{\lambda - \mu}{2}\frac{\hbar}{m\omega} \coth \frac{\hbar\omega}{2kT}, \quad D_{pq} = 0, \quad (4)$$

where T is the temperature of the thermal bath.

In the Markovian regime the harmonic oscillator master equation which satisfies the complete positivity condition cannot satisfy simultaneously the translational invariance and the detailed balance (which assures an asymptotic approach to the canonical thermal equilibrium state). The necessary and sufficient condition for translational invariance is $\lambda = \mu$ [9, 10]. In this case the equations of motion for the expectation values of coordinate and momentum are exactly the same as the classical ones. If $\lambda \neq \mu$, then we violate translational invariance, but we keep the canonical equilibrium state.

The asymptotic values $\sigma_{qq}(\infty), \sigma_{pp}(\infty), \sigma_{pq}(\infty)$ do not depend on the initial values $\sigma_{qq}(0), \sigma_{pp}(0), \sigma_{pq}(0)$ and in the case of a thermal bath with coefficients (4), they reduce to [9, 10]

$$\sigma_{qq}(\infty) = \frac{\hbar}{2m\omega} \coth \frac{\hbar\omega}{2kT}, \quad \sigma_{pp}(\infty) = \frac{\hbar m\omega}{2} \coth \frac{\hbar\omega}{2kT}, \quad \sigma_{pq}(\infty) = 0. \quad (5)$$

In the following, we consider a general temperature T , but we should stress that the Lindblad theory is obtained in the Markov approximation, which holds for high temperatures of the environment. At the same time, the semigroup dynamics of the

density operator which must hold for a quantum Markovian process is valid only for the weak-coupling regime, with the damping λ obeying the inequality $\lambda \ll \omega$.

We consider a harmonic oscillator with an initial Gaussian wave function

$$\Psi(q) = \left(\frac{1}{2\pi\sigma_{qq}(0)}\right)^{\frac{1}{4}} \exp\left[-\frac{1}{4\sigma_{qq}(0)}\left(1 - \frac{2i}{\hbar}\sigma_{pq}(0)\right)(q - \sigma_q(0))^2 + \frac{i}{\hbar}\sigma_p(0)q\right], \quad (6)$$

where $\sigma_{qq}(0)$ is the initial spread, $\sigma_{pq}(0)$ the initial covariance, and $\sigma_q(0)$ and $\sigma_p(0)$ are the initial averaged position and momentum of the wave packet. The initial state (6) represents a correlated coherent state (squeezed coherent state) [11] with the variances and covariance of coordinate and momentum

$$\sigma_{qq}(0) = \frac{\hbar\delta}{2m\omega}, \quad \sigma_{pp}(0) = \frac{\hbar m\omega}{2\delta(1-r^2)}, \quad \sigma_{pq}(0) = \frac{\hbar r}{2\sqrt{1-r^2}}. \quad (7)$$

Here, δ is the squeezing parameter which measures the spread in the initial Gaussian packet and r , with $|r| < 1$ is the correlation coefficient at time $t = 0$. The initial values (7) correspond to a minimum uncertainty state, since they fulfil the generalized uncertainty relation

$$\sigma_{qq}(0)\sigma_{pp}(0) - \sigma_{pq}^2(0) = \frac{\hbar^2}{4}. \quad (8)$$

For $\delta = 1$ and $r = 0$ the correlated coherent state becomes a Glauber coherent state.

From Eq. (3) we derive the evolution equation in coordinate representation:

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & \frac{i\hbar}{2m}\left(\frac{\partial^2}{\partial q^2} - \frac{\partial^2}{\partial q'^2}\right)\rho - \frac{im\omega^2}{2\hbar}(q^2 - q'^2)\rho \\ & - \frac{1}{2}(\lambda + \mu)(q - q')\left(\frac{\partial}{\partial q} - \frac{\partial}{\partial q'}\right)\rho + \frac{1}{2}(\lambda - \mu)\left[(q + q')\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'}\right) + 2\right]\rho \\ & - \frac{D_{pp}}{\hbar^2}(q - q')^2\rho + D_{qq}\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'}\right)^2\rho - 2iD_{pq}\hbar(q - q')\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'}\right)\rho. \end{aligned} \quad (9)$$

For the case of a thermal bath with coefficients (4) the Wigner distribution function $W(q, p, t)$ satisfies the following Fokker-Planck-type equation:

$$\begin{aligned} \frac{\partial W}{\partial t} = & -\frac{p}{m}\frac{\partial W}{\partial q} + m\omega^2 q\frac{\partial W}{\partial p} + (\lambda + \mu)\frac{\partial}{\partial p}(pW) + (\lambda - \mu)\frac{\partial}{\partial q}(qW) \\ & + \frac{\hbar}{2}\coth\frac{\hbar\omega}{2kT}\left[(\lambda + \mu)m\omega\frac{\partial^2 W}{\partial p^2} + \frac{\lambda - \mu}{m\omega}\frac{\partial^2 W}{\partial q^2}\right]. \end{aligned} \quad (10)$$

The first two terms on the right-hand side of both these equations generate a purely unitary evolution. They give the usual Liouvillian evolution. The third and forth terms are the dissipative terms and have a damping effect (exchange of energy with

environment). The last two are noise (diffusive) terms and produce fluctuation effects in the evolution of the system. They promote diffusion in momentum p and coordinate q and generate decoherence in coordinate and momentum, respectively. In the high temperature limit, quantum Fokker-Planck equation (10) with coefficients (4) becomes classical Kramers equation ($D_{pp} \rightarrow 2m\lambda kT$ for $\lambda = \mu$).

The density matrix solution of Eq. (9) has the general form of Gaussian density matrices

$$\begin{aligned} \langle q|\rho(t)|q' \rangle = & \left(\frac{1}{2\pi\sigma_{qq}(t)}\right)^{\frac{1}{2}} \exp\left[-\frac{1}{2\sigma_{qq}(t)}\left(\frac{q+q'}{2} - \sigma_q(t)\right)^2 \right. \\ & \left. -\frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}(q-q')^2 + \frac{i\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)}\left(\frac{q+q'}{2} - \sigma_q(t)\right)(q-q') + \frac{i}{\hbar}\sigma_p(t)(q-q')\right], \end{aligned} \quad (11)$$

where $\sigma(t) \equiv \sigma_{qq}(t)\sigma_{pp}(t) - \sigma_{pq}^2(t)$ is the Schrödinger generalized uncertainty function [12] (σ_{qq} and σ_{pp} denote the dispersion (variance) of the coordinate and momentum, respectively, and σ_{pq} denotes the correlation (covariance) of the coordinate and momentum).

For an initial Gaussian Wigner function (corresponding to a correlated coherent state (6)) the solution of Eq. (10) is

$$\begin{aligned} W(q, p, t) = & \frac{1}{2\pi\sqrt{\sigma(t)}} \exp\left\{-\frac{1}{2\sigma(t)}[\sigma_{pp}(t)(q - \sigma_q(t))^2 + \sigma_{qq}(t)(p - \sigma_p(t))^2 \right. \\ & \left. - 2\sigma_{pq}(t)(q - \sigma_q(t))(p - \sigma_p(t))]\right\}. \end{aligned} \quad (12)$$

In the case of a thermal bath we obtain the following steady state solution for $t \rightarrow \infty$ (we denote $\epsilon \equiv \frac{\hbar\omega}{2kT}$):

$$\langle q|\rho(\infty)|q' \rangle = \left(\frac{m\omega}{\pi\hbar \coth \epsilon}\right)^{\frac{1}{2}} \exp\left\{-\frac{m\omega}{4\hbar}\left[\frac{(q+q')^2}{\coth \epsilon} + (q-q')^2 \coth \epsilon\right]\right\}. \quad (13)$$

In the long time limit we have also

$$W_{\infty}(q, p) = \frac{1}{\pi\hbar \coth \epsilon} \exp\left\{-\frac{1}{\hbar \coth \epsilon}\left[m\omega q^2 + \frac{p^2}{m\omega}\right]\right\}. \quad (14)$$

Stationary solutions to the evolution equations obtained in the long time limit are possible as a result of a balance between the wave packet spreading induced by the Hamiltonian and the localizing effect of the Lindblad operators.

3 Quantum decoherence and classical correlations

As we already stated, one considers that two conditions have to be satisfied in order that a system could be considered as classical. The *first* condition requires that the

system should be in one of relatively permanent states – states that are least affected by the interaction of the system with the environment – and the interference between different states should be negligible. This implies the destruction of off-diagonal elements representing coherences between quantum states in the density matrix, which is the QD phenomenon. The loss of coherence can be achieved by introducing an interaction between the system and environment: an initial pure state with a density matrix which contains nonzero off-diagonal terms can non-unitarily evolve into a final mixed state with a diagonal density matrix during the interaction with the environment, like in classical statistical mechanics.

The *second* condition requires that the system should have, with a good approximation, an evolution according to classical laws. This implies that the Wigner distribution function has a peak along a classical trajectory, that means there exist CC between the canonical variables of coordinate and momentum. Of course, the correlation between the canonical variables, necessary to obtain a classical limit, should not violate Heisenberg uncertainty principle, i.e. the position and momentum should take reasonably sharp values, to a degree in concordance with the uncertainty principle. This is possible, because the density matrix does not diagonalize exactly in position, but with a non-zero width.

Using new variables $\Sigma = (q + q')/2$ and $\Delta = q - q'$, the density matrix (11) can be rewritten as

$$\rho(\Sigma, \Delta, t) = \sqrt{\frac{\alpha}{\pi}} \exp[-\alpha\Sigma^2 - \gamma\Delta^2 + i\beta\Sigma\Delta + 2\alpha\sigma_q(t)\Sigma + i(\frac{\sigma_p(t)}{\hbar} - \beta\sigma_q(t))\Delta - \alpha\sigma_q^2(t)], \quad (15)$$

with the abbreviations

$$\alpha = \frac{1}{2\sigma_{qq}(t)}, \quad \gamma = \frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}, \quad \beta = \frac{\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)} \quad (16)$$

and the Wigner transform of the density matrix (15) is

$$W(q, p, t) = \frac{1}{2\pi\hbar} \sqrt{\frac{\alpha}{\gamma}} \exp\left\{-\frac{[\hbar\beta(q - \sigma_q(t)) - (p - \sigma_p(t))]^2}{4\hbar^2\gamma} - \alpha(q - \sigma_q(t))^2\right\}. \quad (17)$$

a) Degree of quantum decoherence (QD)

The representation-independent measure of the degree of QD [4] is given by the ratio of the dispersion $1/\sqrt{2\gamma}$ of the off-diagonal element $\rho(0, \Delta, t)$ to the dispersion $\sqrt{2/\alpha}$ of the diagonal element $\rho(\Sigma, 0, t)$:

$$\delta_{QD} = \frac{1}{2} \sqrt{\frac{\alpha}{\gamma}}, \quad (18)$$

which in our case gives

$$\delta_{QD}(t) = \frac{\hbar}{2\sqrt{\sigma(t)}}. \quad (19)$$

The finite temperature Schrödinger generalized uncertainty function, calculated in Ref. [12], has the expression

$$\begin{aligned} \sigma(t) = & \frac{\hbar^2}{4} \{ e^{-4\lambda t} [1 - (\delta + \frac{1}{\delta(1-r^2)}) \coth \epsilon + \coth^2 \epsilon] \\ & + e^{-2\lambda t} \coth \epsilon [(\delta + \frac{1}{\delta(1-r^2)} - 2 \coth \epsilon) \frac{\omega^2 - \mu^2 \cos(2\Omega t)}{\Omega^2} \\ & + (\delta - \frac{1}{\delta(1-r^2)}) \frac{\mu \sin(2\Omega t)}{\Omega} + \frac{2r\mu\omega(1 - \cos(2\Omega t))}{\Omega^2 \sqrt{1-r^2}}] + \coth^2 \epsilon \}. \end{aligned} \quad (20)$$

In the limit of long times Eq. (20) yields

$$\sigma(\infty) = \frac{\hbar^2}{4} \coth^2 \epsilon, \quad (21)$$

so that we obtain

$$\delta_{QD}(\infty) = \tanh \frac{\hbar\omega}{2kT}, \quad (22)$$

which for high T becomes

$$\delta_{QD}(\infty) = \frac{\hbar\omega}{2kT}. \quad (23)$$

We see that δ_{QD} decreases, and therefore QD increases, with temperature, i.e. the density matrix becomes more and more diagonal at higher T and the contributions of the off-diagonal elements get smaller and smaller. At the same time the degree of purity decreases and the degree of mixedness increases with T . $\delta_{QD} < 1$ for $T \neq 0$, while for $T = 0$ the asymptotic (final) state is pure and δ_{QD} reaches its initial maximum value 1. $\delta_{QD} = 0$ when the quantum coherence is completely lost. So, when $\delta_{QD} = 1$ there is no QD and only if $\delta_{QD} < 1$, there is a significant degree of QD, when the magnitude of the elements of the density matrix in the position basis are peaked preferentially along the diagonal $q = q'$. When $\delta_{QD} \ll 1$, we have a strong QD.

b) *Degree of classical correlations (CC)*

In defining the degree of CC, the form of the Wigner function is essential, but not its position around $\sigma_q(t)$ and $\sigma_p(t)$. Consequently, for simplicity we consider zero

values for the initial expectations values of the coordinate and momentum and the expression (17) of the Wigner function becomes

$$W(q, p, t) = \frac{1}{2\pi\hbar} \sqrt{\frac{\alpha}{\gamma}} \exp\left[-\frac{(\hbar\beta q - p)^2}{4\hbar^2\gamma} - \alpha q^2\right]. \quad (24)$$

As a measure of the degree of CC we take the relative sharpness of this peak in the phase space determined from the dispersion $\hbar\sqrt{2\gamma}$ in p in Eq. (24) and the magnitude of the average of p ($p_0 = \hbar\beta q$) [4]:

$$\delta_{CC} = \frac{2\sqrt{\alpha\gamma}}{|\beta|}, \quad (25)$$

where we identified q as the dispersion $1/\sqrt{2\alpha}$ of q . δ_{CC} is a good measure of the "squeezing" of the Wigner function in phase space [4]: in the state (24), more "squeezed" is the Wigner function, more strongly established are CC.

For our case, we obtain

$$\delta_{CC}(t) = \frac{\sqrt{\sigma(t)}}{|\sigma_{pq}(t)|}, \quad (26)$$

where $\sigma(t)$ is given by Eq. (20) and $\sigma_{pq}(t)$ can be calculated using formulas given in Refs. [9, 10]:

$$\begin{aligned} \sigma_{pq}(t) = & \frac{\hbar}{4\Omega^2} e^{-2\lambda t} \left\{ \left[\mu\omega(2\coth\epsilon - \delta - \frac{1}{\delta(1-r^2)}) - \frac{2\omega^2 r}{\sqrt{1-r^2}} \right] \cos(2\Omega t) \right. \\ & \left. + \omega\Omega(\delta - \frac{1}{\delta(1-r^2)}) \sin(2\Omega t) + \mu\omega(\delta + \frac{1}{\delta(1-r^2)} - 2\coth\epsilon) + \frac{2\mu^2 r}{\sqrt{1-r^2}} \right\}. \end{aligned} \quad (27)$$

When δ_{CC} is of order of unity, we have a significant degree of classical correlations. The condition of strong CC is $\delta_{CC} \ll 1$, which assures a very sharp peak in phase space. Since $\sigma_{pq}(\infty) = 0$, in the case of an asymptotic Gibbs state, we get $\delta_{CC}(\infty) \rightarrow \infty$, so that our expression shows no CC at $t \rightarrow \infty$.

c) Discussion with Gaussian density matrix and Wigner function

If the initial wave function is Gaussian, then the density matrix (11) and the Wigner function (12) remain Gaussian for all times (with time-dependent parameters which determine their amplitude and spread) and centered along the trajectory given by the solutions of the dissipative equations of motion. This trajectory is exactly classical for $\lambda = \mu$ and only approximately classical for not large $\lambda - \mu$.

The degree of QD has an evolution which shows that in general QD increases with time and temperature. The degree of CC has a more complicated evolution,

but the general tendency is that CC are less and less strong with increasing time and temperature. $\delta_{QD} < 1$ and δ_{CC} is of the order of unity for a long enough interval of time, so that we can say that the considered system interacting with the thermal bath manifests both QD and CC and a true quantum to classical transition takes place. Dissipation promotes quantum coherences, whereas fluctuation (diffusion) reduces coherences and promotes QD. The balance of dissipation and fluctuation determines the final equilibrium value of δ_{QD} . The quantum system starts as a pure state, with a Wigner function well localized in phase space (Gaussian form). This state evolves approximately following the classical trajectory (Liouville flow) in phase space and becomes a quantum mixed state during the irreversible process of QD.

d) *Decoherence time*

In order to obtain the decoherence time, we consider the coefficient γ (16), which measures the contribution of non-diagonal terms in the density matrix (15). For short times ($\lambda t \ll 1, \Omega t \ll 1$), we have:

$$\gamma(t) = -\frac{m\omega}{4\hbar\delta} \left\{ 1 + 2\left[\lambda\left(\delta + \frac{r^2}{\delta(1-r^2)}\right) \coth \epsilon + \mu\left(\delta - \frac{r^2}{\delta(1-r^2)}\right) \coth \epsilon - \lambda - \mu - \frac{\omega r}{\delta\sqrt{1-r^2}}\right] t \right\}. \quad (28)$$

From here we obtain that quantum coherences in the density matrix decay exponentially at a rate given by

$$2\left[\lambda\left(\delta + \frac{r^2}{\delta(1-r^2)}\right) \coth \epsilon + \mu\left(\delta - \frac{r^2}{\delta(1-r^2)}\right) \coth \epsilon - \lambda - \mu - \frac{\omega r}{\delta\sqrt{1-r^2}}\right] \quad (29)$$

and then the decoherence time scale is

$$t_{deco} = \frac{1}{2\left[\lambda\left(\delta + \frac{r^2}{\delta(1-r^2)}\right) \coth \epsilon + \mu\left(\delta - \frac{r^2}{\delta(1-r^2)}\right) \coth \epsilon - \lambda - \mu - \frac{\omega r}{\delta\sqrt{1-r^2}}\right]}. \quad (30)$$

The decoherence time depends on the temperature T and the coupling λ (dissipation coefficient) between the system and environment (through the diffusion coefficient D_{pp}), on the squeezing parameter δ that measures the spread in the initial Gaussian packet and on the initial correlation coefficient r . We notice that the decoherence time is decreasing with increasing dissipation, temperature and squeezing.

For $r = 0$ we obtain:

$$t_{deco} = \frac{1}{2(\lambda + \mu)(\delta \coth \epsilon - 1)} \quad (31)$$

and at temperature $T = 0$ (then we have to take $\mu = 0$), this becomes

$$t_{deco} = \frac{1}{2\lambda(\delta - 1)}. \quad (32)$$

We see that when the initial state is the usual coherent state ($\delta = 1$), then the decoherence time tends to infinity. This corresponds to the fact that for $T = 0$ and $\delta = 1$ the coefficient γ is constant in time, so that the decoherence process does not occur in this case.

At high temperature, expression (30) becomes (we denote $\tau \equiv \frac{1}{\epsilon}$)

$$t_{deco} = \frac{1}{2[\lambda(\delta + \frac{r^2}{\delta(1-r^2)}) + \mu(\delta - \frac{r^2}{\delta(1-r^2)})]\tau}. \quad (33)$$

If, in addition $r = 0$, then we obtain

$$t_{deco} = \frac{\hbar\omega}{4(\lambda + \mu)\delta kT}. \quad (34)$$

In Ref. [12] we determined the time t_d when thermal fluctuations become comparable with quantum fluctuations:

$$t_d = \frac{1}{2[\lambda(\delta + \frac{1}{\delta(1-r^2)}) \coth \epsilon + \mu(\delta - \frac{1}{\delta(1-r^2)}) \coth \epsilon - 2\lambda]}. \quad (35)$$

At high temperature, expression (35) becomes

$$t_d = \frac{1}{2\tau[\lambda(\delta + \frac{1}{\delta(1-r^2)}) + \mu(\delta - \frac{1}{\delta(1-r^2)})]}. \quad (36)$$

As expected, the decoherence time t_{deco} has the same scale as the time t_d after which thermal fluctuations become comparable with quantum fluctuations.

We can assert that in the considered case classicality is a temporary phenomenon, which takes place only at some stages of the dynamical evolution, during a definite interval of time [13]. Due to the dissipative nature of evolution, the approximately deterministic evolution is no more valid for very large times, when the localization of the system is affected by the spreading of the wave packet and of the Wigner distribution function.

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